Refresher course: Numerical Matrix Analysis & Optimization

Lecturers: from the applied maths and computer science lab LJK

Jérôme Malick  CNRS researcher

Franck Iutzeler  Assistant Professor

jerome.malick@imag.fr
franck.iutzeler@imag.fr

Students: 43 Master2 students

▶ Master Computer Sciences MoSiG
▶ Master of Applied Maths MSIAM
▶ Other Masters (MiSCIT, ORCO)

▶ From around the world: Europe, Russia, India, China, Africa, ...
▶ diverse origins and backgrounds : need for basic commun knowledge
Why matrix analysis and optimization?

Matrix and optimization are at the heart of computational mathematics.

With applications everywhere, e.g.

- Machine Learning
- Energy Management
- Signal/Image Processing

Mix between theory and practice:

**Theory**
- Meaning of a problem;
- Existence/uniqueness of solution; math properties

**Practice**
- Computability, speed, and use of standard libraries to solve numerically these problems.

This short course focuses on matrix analysis and optimization in action with exercises inspired from:

- Google PageRank, Image processing
- Machine learning applications (regression, classification)
This course is not a standard course on linear algebra or optimization

- not a math course
  basic knowledge is assumed (take a look to your undergraduate courses)
- not an algorithmic course
  basic programming skills are expected (check-out Python tutorials)

This course is

- a review of basics of matrix analysis – from numerical perspective
- a short overview of numerical optimization
- includes quick recalls from matrix calculus and differential calculus
Organisation

Schedule: three dense days (before the other courses start)

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<th>9:45 – 12:45</th>
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<tr>
<td>Wed 21st</td>
<td>Intro &amp; Course on Matrices Amphi H</td>
<td>Tutorial on Matrices Amphi H</td>
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<td>Thu 22nd</td>
<td>Lab. on Matrices E301</td>
<td>Course on Optimization Amphi D</td>
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<tr>
<td>Fri 23rd</td>
<td>Tutorial on Optimization Amphi H</td>
<td>Lab. on Optimization E103</td>
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Tutorials:
- to manipulate the notions on simple examples
  no fancy maths and no computations - they will be done on machines
- prepare the labs sessions...

Labs.:
- matrices and optimization in action for learning and ranking
- with Python & Numpy, by groups of 1 or 2
  Take your login or your own machine (with iPython installed!)
Course 1 on matrix analysis

1. Basics on matrices
   - Matrices and operations between matrices
   - Operations on matrices: transpose, trace, det
   - Special matrices (triangular, symmetric, orthogonal, invertible, SDP)
   - Decomposition: (P)LU, QR

2. Linear systems
   - Invertible systems, linear least-squares, linear least-norm
   - Easy systems for special matrices (triangular, orthogonal,...)
   - Solving systems: by factorization, by iterative methods, by optimization
   - Practical considerations (preconditioning, software,...)

3. Spectral decompositions
   - Eigenvalues: real, complex, spectral radius
   - Eigenvalue decomposition, geometric interpretation
   - Singular value decomposition: SVD, compact SVD, link with eigenvalues

+ Note on matrix norms: standard norms, induced/operator norms, connection with spectral radius
Course 2 on numerical optimization

1. Introduction: what is optimization?
   - Optimization problems: definitions, examples, first properties
   - How to solve an optimization problems: exact/approximate solutions, difficult/impossible in general, "easy" for linear... and convex problems
   - A classification: cvx/non-cvx, smooth/non-smooth, stochastic/deterministic

2. Convexity and optimization
   - Convex sets and functions, examples
   - Convex optimization problems: global solutions, convex set of solutions
   - Recognizing convexity: definition, convexity-preserving operations, Hessian
   - In practice: modeling, interface, algorithms, experience

3. Simple algorithm for a simple problem: gradient algorithm
   - Unconstrained convex differentiable problems, optimality conditions
   - Gradient algorithm with 4 ingredients of all algorithms
   - Study: convergence theorem vs numerical experiments
   - Beyond gradient: acceleration, 2nd order, Newton

Notes:
- Recalls on derivatives: gradient, Hessian, chain rule, examples
- Application to classification: geometrical/statistical problems, optim. models
This course introduces **material** for several courses, among them:

- "Efficient methods in optimization"  
  (on convex analysis & complexity and convergence of algorithms)
- "Convex and distributed optimization"  (for large-scale applications & big data)
- 3 courses on machine learning!
- PDEs, inverse problems, stats...

**Useful Links:**

- Python/Numpy’s documentation  
  [http://docs.scipy.org/doc/numpy-1.11.0/reference/](http://docs.scipy.org/doc/numpy-1.11.0/reference/)
- Stephen’s Boyd website (check the courses, quizzes, and exercises)  

**Main References:**

- Ciarlet, Ph.: *Introduction à l’analyse numérique et l’optimisation*.